ELC5423-THEORETICAL FOUNDATIONS OF ELECTRICAL ENGINEERING
6B071101 – POWER ENGINEERING

Analytical representation of vectors. Addition and subtraction of sinusoidal functions.
Ohm's Law in complex form.

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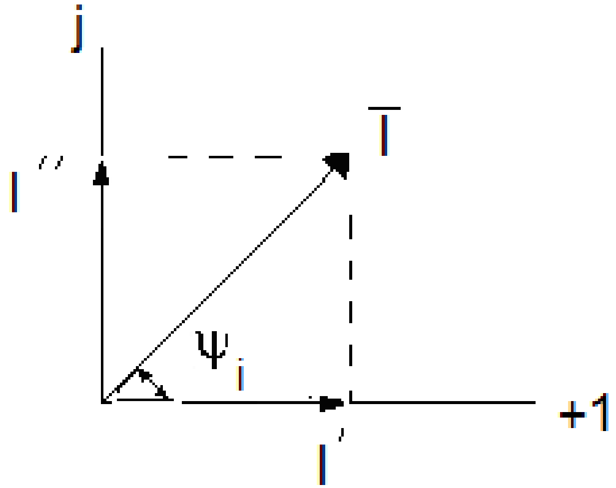
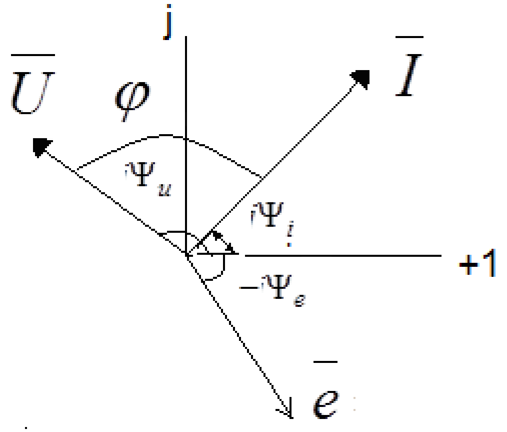
Vector - complex number

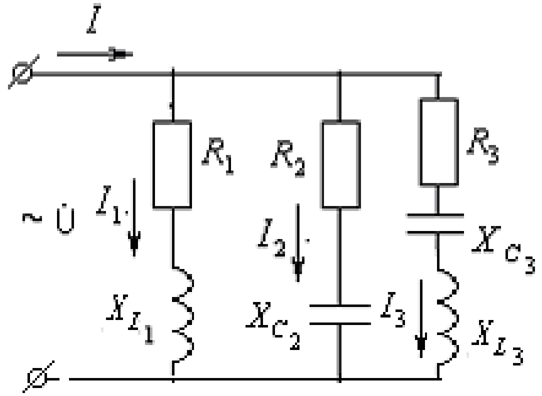
In an electrical circuit with parallel connection of receivers, the voltages on each branch are equal, and the total current is equal to the sum of the branch currents. Such an electrical circuit can be calculated by graph analytic method. The current in each branch is determined by Ohm's law:

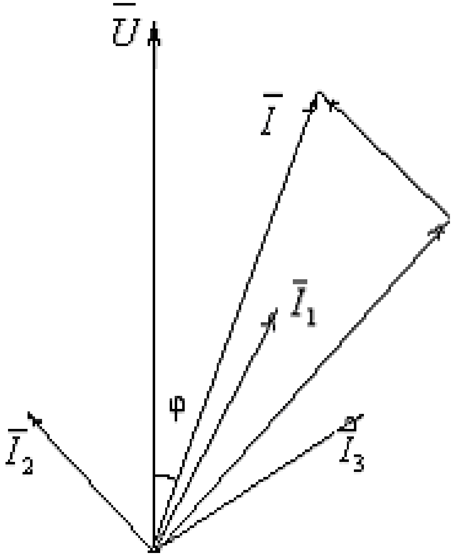


$$\bar{I} = I' + jI''$$



$$\Psi_u - \Psi_i = \phi$$





$$I_1 = \frac{U}{\sqrt{R_1^2 + X_{L_1}^2}} \qquad I_2 = \frac{U}{\sqrt{R_2^2 + X_{C_2}^2}} \qquad I_3 = \frac{U}{\sqrt{R_3^2 + (X_{L_3} - X_{C_3})^2}}$$

The angle of shift between current and voltage in each branch is determined using:

$$\cos\varphi_1 = \frac{R_1}{\sqrt{R_1^2 + X_{L_1}^2}} \qquad \cos\varphi_2 = \frac{R_2}{\sqrt{R_2^2 + X_{C_2}^2}} \qquad \cos\varphi_3 = \frac{R_3}{\sqrt{R_3^2 + (X_{L_3} - X_{C_3})^2}}$$

Euler's formula is a fundamental equation in complex analysis and trigonometry, given by:

$$e^{ix}=\cos(x)+i\sin(x)$$

where:

- ***e*** **is the base of the natural logarithm,**
- ***i*** **is the imaginary unit** ($i^2=-1$),
- ***x*** **is a real number, typically an angle in radians.**

Euler's formula provides a link between exponential functions and trigonometric functions. For a complex number with modulus r and angle θ, the exponential form is given by:

$$z = re^{i\theta} = r(\cos(\theta) + i\sin(\theta))$$

This expression is helpful because it simplifies the representation and manipulation of complex numbers, especially in contexts involving rotation, waves, and oscillations.

It is possible to represent $\boldsymbol{z}$ in polar coordinates by determining: $r = |z| = \sqrt{x^2 + y^2}$

Angle (argument) $\boldsymbol{\theta}$, given by: $\theta = \arg(z) = \tan^{-1}\left(\frac{y}{x}\right)$

Then the trigonometric form of $\boldsymbol{z}$ is: $z = r(\cos\theta + i\sin\theta)$

Exponential Form

Using **Euler's formula**, which states:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

we can rewrite the trigonometric form of ***z*** in exponential form as:

$$z = re^{i\theta}$$

where r is the magnitude and $\boldsymbol{\theta}$ is the angle of the complex number.

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Example

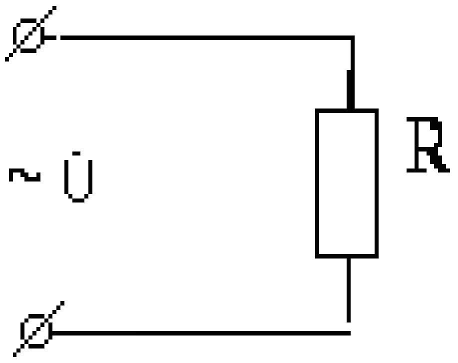
If $z = 3 + 4i$:

1. **Magnitude** $r = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = 5$.
2. **Angle** $\theta = \tan^{-1}\left(\frac{4}{3}\right) \approx 0.93$ radians.

Thus:

- **Trigonometric Form:** $z = 5(\cos 0.93 + i \sin 0.93)$
- **Exponential Form:** $z = 5e^{i \cdot 0.93}$

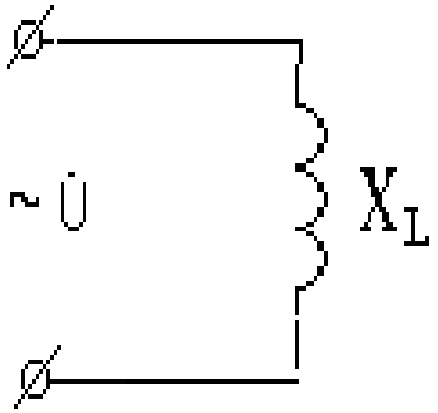
Ohm’s law in complex form



$$\frac{\overline{U}}{\overline{I}} = \frac{Ue^{j\Psi_u}}{Ie^{j\Psi_i}} = \frac{U}{I} e^{j(\Psi_u - \Psi_i)} = \mathrm{R}e^{j\varphi} = \mathrm{R}e^{j0} = R$$

$$\overline{R} = R$$

$$\overline{I} = \frac{\overline{U}_R}{R}$$

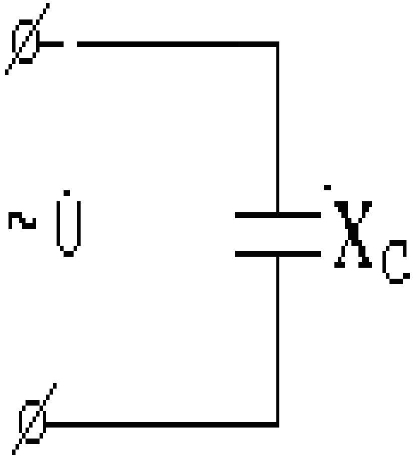


$$\frac{\overline{U}}{\overline{I}} = \frac{Ue^{j\Psi_u}}{Ie^{j\Psi_i}} = \frac{U}{I}e^{j(\Psi_u - \Psi_i)} = X_L e^{j\varphi} = X_L e^{j90^0} = jX_L$$

$$e^{j90^0} = \cos 90^0 + j\sin 90^0 = j$$

$$\overline{X_L} = jX_L$$

$$\overline{I} = \frac{\overline{U}_L}{\overline{X}_L}$$

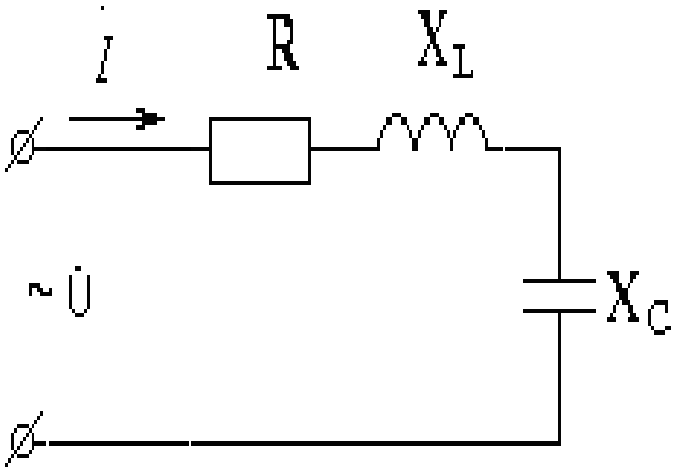


$$\frac{\overline{U}}{\overline{I}} = \frac{Ue^{j\Psi_u}}{Ie^{j\Psi_i}} = \frac{U}{I} e^{j(\Psi_u - \Psi_i)} = X_C e^{j\varphi} = X_C e^{-j90^0} = -jX_C$$

$$e^{-j90^0} = \cos(-90^0) + j\sin(-90^0) = -j$$

$$\overline{X_C} = -jX_C$$

$$\overline{I} = \frac{\overline{U}_C}{\overline{X}_C}$$



$$\overline{U} = \overline{U}_R + \overline{U}_L + \overline{U}_C$$

$$\overline{U}_R = \overline{I}R$$

$$\overline{U}_L = j\overline{I}X_L$$

$$\overline{U}_C = -j\overline{I}X_C$$

$$\overline{U} = \overline{I}\left[R + j\left(X_L - X_C\right)\right]$$

$$\overline{I} = \frac{\overline{U}}{\overline{Z}}$$

$$\overline{Z} = \left[R + j\left(X_L - X_C\right)\right]$$

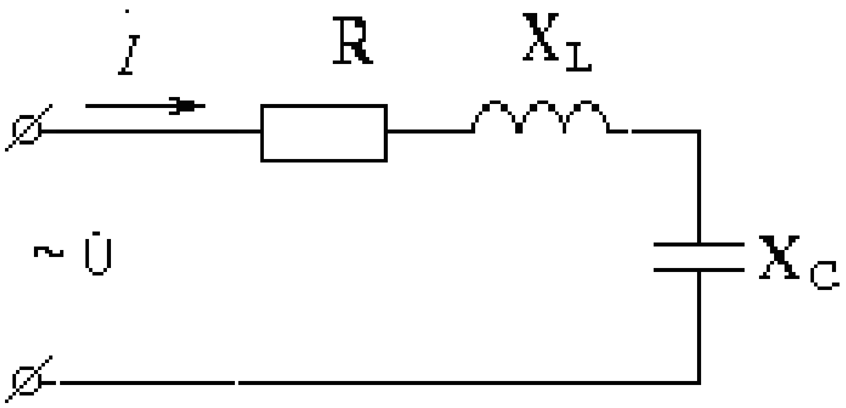
$$\overline{Z} = \frac{\overline{U}}{\overline{I}} = \frac{Ue^{j\Psi_u}}{Ie^{j\Psi_i}} = Ze^{j(\Psi_u - \Psi_i)} = Ze^{j\varphi}$$

Given , . F

$$\bar{I} = \frac{\bar{U}}{\bar{Z}}$$

$$I = 25e^{j73°}/5e^{j50°} = 5e^{j(73°-50°)} = 5e^{j23°}$$

Determine a complex resistance of electrical circuit if: R =8 Ohm , X_L =10 Ohm, X_C =16 Ohm



$$\overline{Z} = \left[R + j\left(X_L - X_C\right)\right]$$

$$Z=8-j6=82+62e^{-jarctg68}=10e^{-j370}$$